A Method of Measuring Acoustic Impedance *

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An apparatus is described whereby acoustic impedances may be measured in terms of a known acoustic impedance and the complex ratios of two electrical potentiometer readings to a third. As a known impedance, there is chosen the reactance of a closed tube of uniform bore which is an eighth wave-length long. The electrical readings are obtained by balancing the amplified output of a condenser transmitter against the electrical input of the source of sound. The condenser transmitter picks up the acoustic pressure at the junction of the sound-source and the attached impedance. A balance is made for each of three successively attached impedances: (1) a closed tube an eighth wave-length long, (2) a rigid closure of the sound-source, and (3) the impedance to be measured. The unknown acoustic impedance Z is then calculated in terms of the known acoustic impedance Z_0 by means of the equation $Z = Z_0 \frac{z_1 - z_2}{z_3 - z_2}$, where z_1, z_2 and z_3 are, respectively, the three electrical impedance settings of the potentiometer. As indicated by this equation, the constants of the electrical circuit are involved only as ratios, so that the response characteristics of the source of the sound, condenser transmitter and amplifiers (provided they are invariable) do not affect the measurement.

Illustrations are given of impedance measurements on a closed tube of uniform bore, a conical horn, an exponential horn, an " infinite " tube, and a hole in an " infinite " wall.

THE progress in acoustics during the past few years has caused acoustic impedance measurement to have the same relative importance that impedance measurement in electrical work has had for many years. The concept of acoustic impedance is derived from the analogy [1] that exists between electrical and acoustic devices, as shown by the analogous differential equations describing their action. Acoustic impedance is usually defined as the complex ratio of pressure to volume velocity (or flux) but it is sometimes more convenient to deal with ratios of pressure to linear velocity or force to linear velocity. The magnitudes of these are interrelated, of course, by powers of the area involved.

The earliest efforts to measure acoustic impedance seem to have been made by Kennelly and Kurokawa.[2] In their method, electrical measurements were made of the motional impedance of a telephone receiver, with and without an attached acoustic impedance. Except for frequencies near resonance, the method was inaccurate because the acoustic impedance was associated with a relatively large mechanical impedance.

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[1] This analogy was first pointed out by A. G. Webster in *Nat. Acad. of Science*, **5**, 275 (1919).

[2] *Proc. Am. Ac. Arts and Sc.*, **56**, 1 (1921).

Later, a direct method was described by Stewart [3] who measured the change in acoustic transmission through a long uniform tube when the unknown impedance was inserted as a branch.

The apparatus to be described in this paper measures acoustic impedance directly in terms of a known acoustic impedance and three balance readings of an electrical potentiometer. The only assumptions involved in the method are that the elements of the apparatus be invariable during a measurement, and that the value of the comparison acoustic impedance be known accurately.

Apparatus

Fig. 1 shows the general arrangement of the apparatus. An oscillator feeds electrical energy into a loud speaker where a portion is converted into acoustic energy which travels along the tube and into an

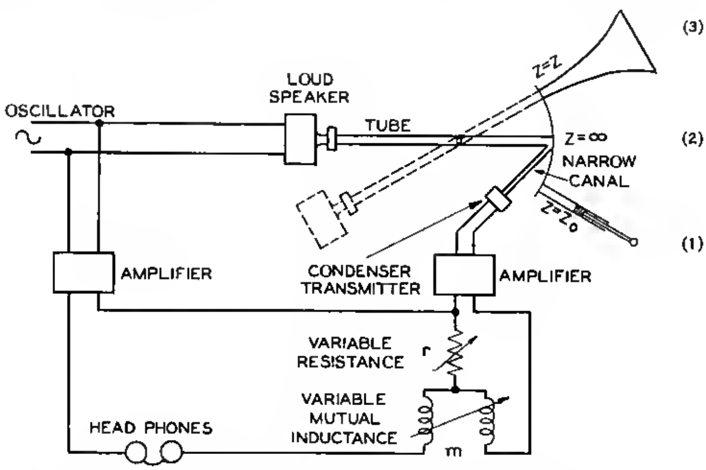

Fig. 1—Schematic circuit of acoustic impedance measuring apparatus.

attached impedance. A canal about 0.06 inches in diameter picks up the sound pressure at the junction of the tube and the attached impedance, and passes it along to a small condenser transmitter. A corresponding voltage, generated by the transmitter, is amplified and the current output of the amplifier passed through a variable resistance in series with the primary of a variable mutual inductance.

The same oscillator also feeds energy through a second amplifier at the output of which the voltage is balanced (by the null method) against the voltage drop across the variable resistance and the secondary of the mutual inductance.

[3] *Phys. Rev.*, **28**, 1038 (1926).

At the end of the tube from the loud speaker are three different impedances. One is the reactance offered by a closed tube of uniform bore. The closure is formed by a well-fitting plunger whose position in the tube may be adjusted. The second is the infinite impedance offered by a rigid wall closing the end of the tube from the loud speaker. The third is the impedance to be measured. All three impedance elements are fixed in position. The loud speaker, tube, condenser transmitter and associated amplifiers are, however, mounted together on a carriage which can be rotated so as to bring any one of these impedances into alignment with the tube. For brevity, reference to these three positions will hereinafter be to positions 1, 2, and 3.

For any one frequency a balance is obtained for each of the three positions. These three electrical readings and the reactance value of the closed tube are sufficient to determine the impedance being measured.

A photograph of the apparatus is shown in Fig. 2.

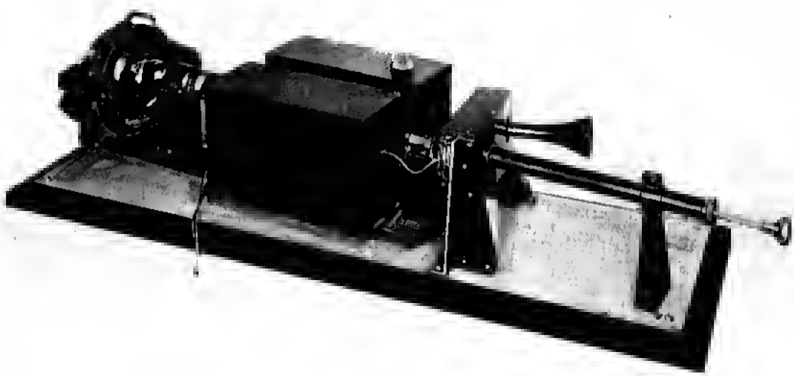

Fig. 2—View of acoustic impedance equipment, showing loud speaker, tubes, condenser transmitter amplifiers and small horn in position for measurement.

Theory

Thevenin's theorem [4] states that, in an invariable electrical network, the current in any branch is equal to the current that would flow in a simple series circuit composed of an electromotive force and two impedances. The electromotive force is the voltage that would obtain at the branch terminals on open circuit. The impedances are the impedance at the terminals looking back into the source of power, and the impedance of the branch.

[4] K. S. Johnson's "Transmission Circuits for Telephone Communication," Ch VIII.

Since the differential equations of acoustics are analogous to those of electrical lines and networks, the theorem may be applied to the action of this apparatus with a considerable saving in labor.[5]

By Thevenin's theorem then, the tube, loud speaker, oscillator, etc. may be replaced by one pressure and one impedance. The pressure is the "open-circuit" pressure at the end of the tube, or in other words the pressure that would be exerted on a rigid wall if placed there. The impedance is the complex ratio of pressure to velocity at the end of the tube which would exist if acoustic energy were sent into it toward the loud speaker, the oscillator being shut off. In electrical terms, this would be called the impedance looking into the source. The velocity or acoustic current that flows into an impedance attached to the end is then the current that would flow in an analogous circuit composed of this vibromotive force or pressure and the two impedances in series. This impedance diagram is given in Fig. 3.

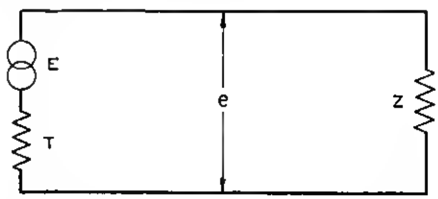

Fig. 3—Impedance diagram for Thevenin's theorem.

E is the open-circuit voltage or pressure, T the impedance looking into the source of sound at the junction, and Z the attached impedance. The pressure e at the junction of T and Z is, of course, the velocity-current $\frac{E}{T+Z}$ in the loop, multiplied by Z. The three equations for three values of attached impedance are

$$Z = Z_0, \qquad e_1 = \frac{EZ_0}{T+Z_0},$$

$$Z = \infty, \qquad e_2 = E,$$

$$Z = Z, \qquad e_3 = \frac{EZ}{T+Z}.$$

The two unknown quantities, E and T, can be eliminated giving one equation

$$\frac{Z}{Z_0} = \frac{\dfrac{e_2}{e_1} - 1}{\dfrac{e_2}{e_3} - 1},$$

[5] A more direct proof of Thevenin's theorem as applied to acoustics is given by W. P. Mason in *B. S. T. J.*, 6, 291 (1927).

whereby Z may be calculated in terms of Z_0 and two ratios of pressures at the junction.

Referring now to Fig. 1 it will be seen that the current through the resistance and mutual primary is proportional to the pressure at the junction. The drop in voltage across the secondary and the resistance is equal in magnitude and opposite in phase to a voltage proportional to E, when no current passes through the head-phones. If k signifies the circuit constant and if z be the impedance value of the resistance and the mutual inductance, then $ez = kE$; and the above equation becomes

$$\frac{Z}{Z_0} = \frac{\dfrac{z_1}{z_2} - 1}{\dfrac{z_3}{z_2} - 1} = \frac{z_1 - z_2}{z_3 - z_2} = \frac{(r_1 - r_2) + j\omega(m_1 - m_2)}{(r_3 - r_2) + j\omega(m_3 - m_2)},$$

where r is the resistance component of z and m is the mutual inductance.

The reactance of a closed tube of uniform bore whose length is one-eighth the wave length of sound for the measuring frequency is chosen as the known impedance. If dissipation in the tube be neglected, the impedance is readily calculated [6] to be a pure negative reactance of 41 mechanical ohms per square centimeter [7] at a temperature of 20° C. This value is chosen because it is of the same order of magnitude as most acoustic impedances. By mechanical ohms per square centimeter is meant the complex ratio of pressure to the linear velocity of the air. The justification for assuming negligible dissipation will be apparent when measurements made on a closed tube, several wave-lengths long, are described.

In making measurements, the three impedance values necessary for balance are read for the three impedance conditions in the 2-1-3 or 2-3-1 order. Afterwards, as a check to ensure that the circuit constant has not changed during the measurement, condition 2 is measured again. This series of four measurements is repeated for each frequency.

Application

Fig. 4 shows the results of measuring the reactance of a closed tube. The tube was 2.4 inches long and 0.7 inch in diameter. The comparison impedance was the calculated reactance of this same tube in the one-eighth wave-length condition, assuming no dissipation. The impedance was also calculated,[8] taking into account viscosity and

[6] See I. B. Crandall's "Theory of Vibrating Systems and Sound," p. 104.

[7] See definitions 8007 and 8011 in "Standardization Report of I. R. E." in "Year Book of I. R. E.," 1931.

[8] See Rayleigh, "Theory of Sound," Vol. II, pp 318 and 325.

losses through heat conduction, for frequencies near the half wave-length anti-resonance, where dissipative effects are most pronounced. It will be seen from Fig. 5 that there is a close agreement between the theoretical curve and the measured points. It seems reasonable, therefore, to assume that the value chosen for the comparison impedance is quite accurate.

Fig. 6 shows the impedance of a conical horn and Fig. 7 that of an exponential horn. In both cases, the mouth of the horn projected through a window into open air, so as to minimize reflection effects

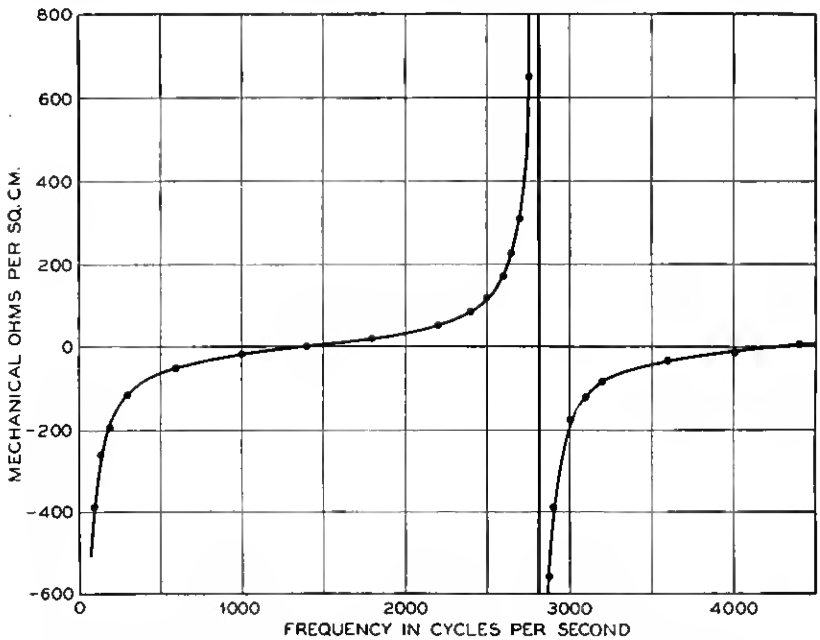

Fig. 4—Acoustic reactance of closed cylindrical tube, 2.4 inches long and 0.7 inch in diameter.

from external objects. Reflection effects from the mouth, where there is a change in impedance, are present, however, and these appear as oscillations of the impedance about a mean which is the characteristic impedance of the horn. By characteristic impedance is meant the impedance that would obtain looking into the throat of the horn were it infinite in length.

Fig. 8 is the impedance of an "infinite" tube. The tube was actually 112 feet long and coiled into a helix. At low frequencies, where the dissipative losses are small, reflection effects from the open end are

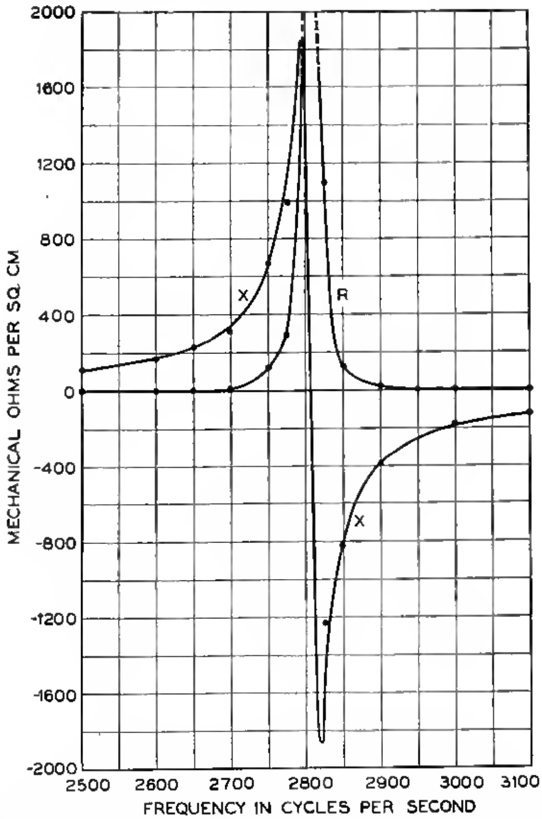

Fig. 5—Acoustic impedance of closed cylindrical tube, 2.4 inches long and 0.7 inch in diameter, showing agreement between measured and calculated values.

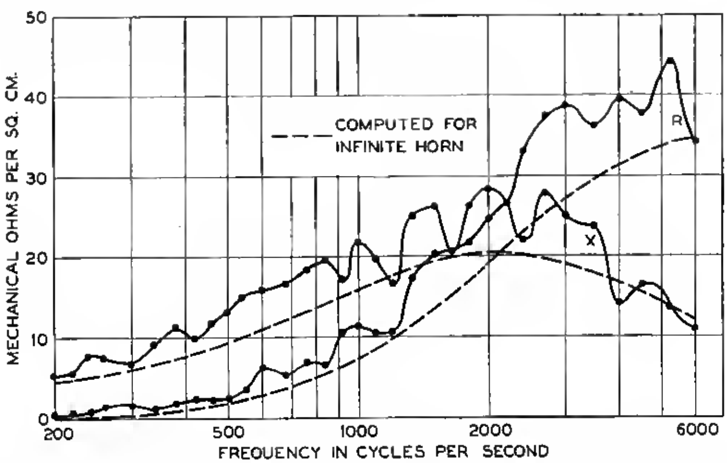

Fig. 6—Acoustic impedance of 38 inch conical horn, having end diameters of 0.7 inch and 28 inches.

observed as oscillations of the impedance at about 5-cycle intervals. An examination of the measurements in this oscillatory region (Fig. 9) will make evident the precision of the apparatus.

Fig. 10 shows the radiation impedance of a hole, 0.7 inch in diameter and surrounded by a flange which approximates an infinite wall for

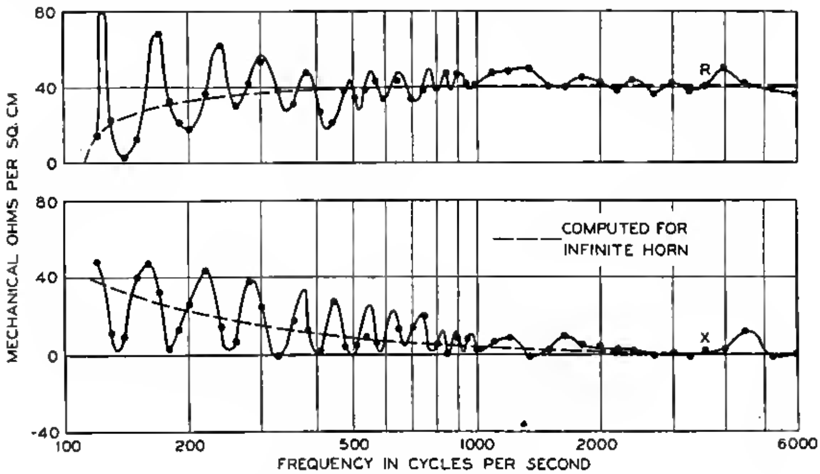

Fig. 7—Acoustic impedance of 6 foot exponential horn, having end diameters of 0.7 inch and 30 inches.

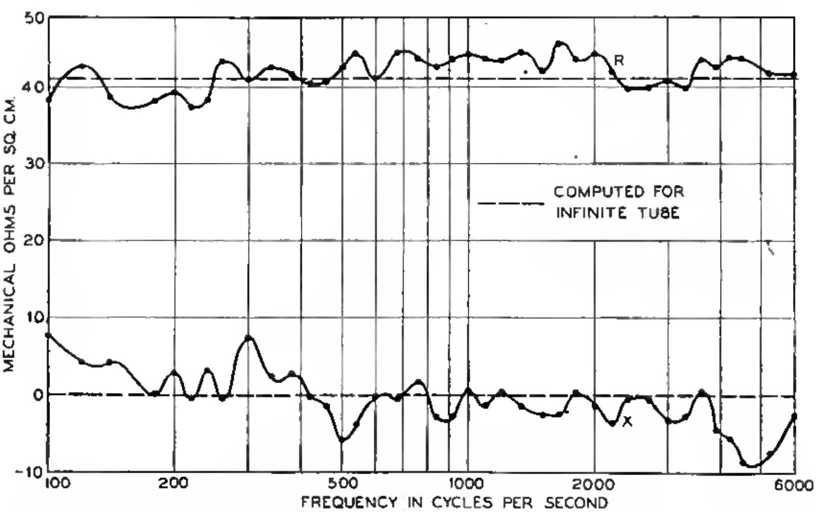

Fig. 8—Acoustic impedance of 112 foot open tube, 0.7 inch in diameter, coiled into helix. Measuring frequencies chosen at random.

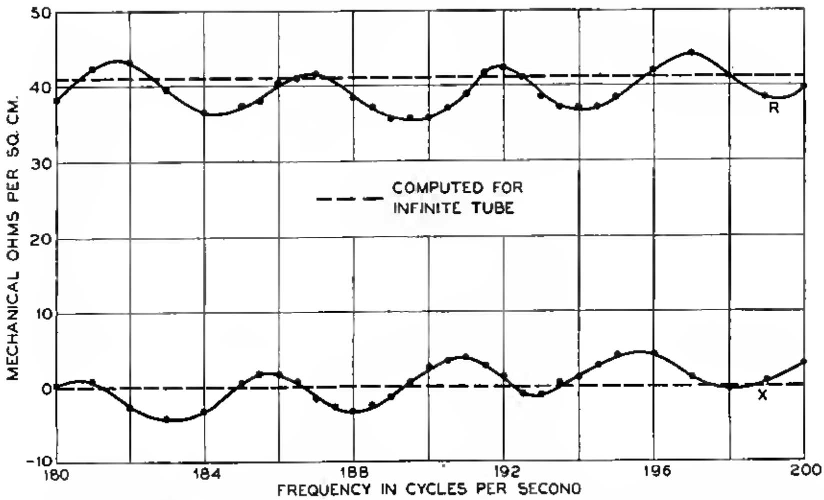

Fig. 9—Acoustic impedance of 112 foot open tube, 0.7 inch in diameter, coiled into helix. Measuring frequencies chosen at half and one cycle intervals to show oscillatory character of impedance.

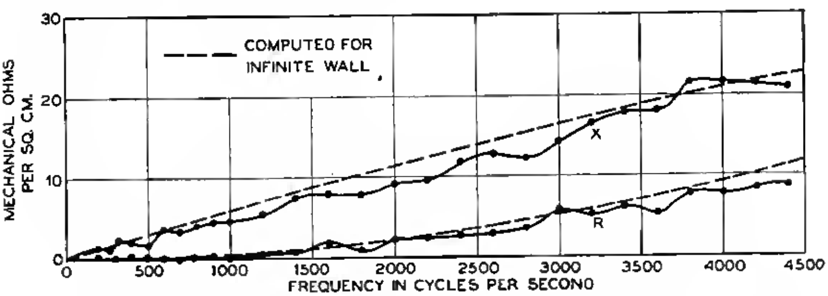

Fig. 10—Acoustic radiation impedance of hole, 0.7 inch in diameter, in flange having diameter of 6 inches.

the frequencies of interest. The dotted lines are the resistance and reactance as calculated by the equations of Rayleigh.[9]

[9] "Theory of Sound," Vol. II, p. 164.